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The return to isotropy of an homogeneous turbulence having been submitted to two successive plane strains

By J. N. GENCE AND J. MATHIEU

Laboratoire de Mécanique des Fluides, Ecole Centrale de Lyon
and Université Claude Bernard – Lyon I

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The authors consider an homogeneous non-isotropic turbulence which develops without mean velocity gradient so that it should return to isotropy. This turbulence has been obtained by application of two successive plane strains to a grid-generated turbulence, and this configuration has already been described in a preceding paper. It is shown in particular that the nonlinear effects make no significant contribution to the rotation of the principal axes of the Reynolds stress tensor. In the case of the return to isotropy, an important parameter connected with the turbulent energy distribution between three directions comes into play. In the present experiment it has a positive sign whereas in previous experiments this sign was negative. In particular, the authors conclude that, when this parameter is positive, the return to isotropy is slower than in the opposite case.

1. Introduction

In a preceding paper (Gence & Mathieu 1979), the authors have considered the action of two successive plane strains on an initially isotropic turbulence. The first strain was used for obtaining an homogeneous but non-isotropic turbulence in which the principal axes of the Reynolds stress tensor are aligned with those of the strain. The second strain, the principal axes of which are different from those of the first strain through an angle α, was applied to this 'oriented' turbulence so that the Reynolds stress tensor should not possess the same principal axes as the ones of the strain. The distorting duct in which these two successive plane strains are obtained has been designed in the same way as in the experiment of Tücker & Reynolds (1968) but the initial cross-section is an ellipse and it is easy to show that there exists downstream a circular cross-section. The duct can then be cut into two parts at this section; accordingly the second part in which the second strain takes place can be turned by an angle α about the X_1 axis as indicated in figure 1. Therefore two successive plane strains of same intensity but with different principal axes are realized. In particular it has been shown that, in the second strain, the principal axes of the Reynolds stress tensor which were initially aligned with those of the first strain, had a tendency to be realigned with those of the second strain. Moreover when the initial angle between the principal axes of the Reynolds stress tensor and those of the second strain was greater than $\frac{1}{4}\pi$, it has been observed that during a finite time the fluctuating motion gave energy to the mean one and that this phenomenon was linked to a forced decay of the anisotropy of the turbulence.

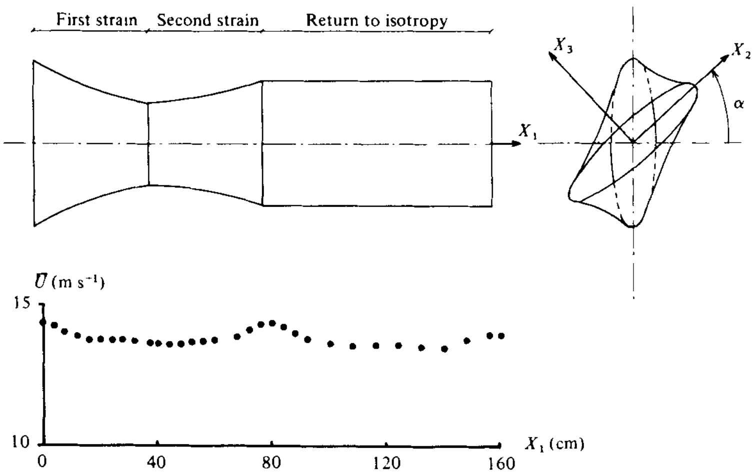

FIGURE 1. The distorting duct and the duct in which the return to isotropy takes place and the mean velocity $\bar{U}_1$ along the X_1 axis.

All these results concern a physical case in which the principal axes of the Reynolds stress tensor are not aligned with those of the strain. This situation leads to a typical aspect of the interaction between the mean flow and the fluctuating motion. In such an experimental situation the dynamic of the turbulence depends not only on the interaction of the mean motion with the fluctuating motion but also on the self-interaction of the fluctuating motion. The unique way to obtain information on both the nonlinear effects and on the terms which are linked to these mechanisms is to suddenly suppress the second strain, so that the turbulence can develop downstream without a mean velocity gradient. This suppression has experimentally been achieved as in the experiment of Tücker & Reynolds (1968), by adding to the two successive plane strains a non-distorting duct 0·8 m in length (figure 1).

The classical phenomenon of the return to isotropy can be observed in the non-distorting duct. Only a few experiments deal with this case, in particular those of Uberoi (1956), Mills & Corrsin (1959) and more recently of Tücker & Reynolds (1968). In the author's experiment, the history of the turbulent motion has been influenced by the action of the two successive plane strains so that we could expect some new phenomena and obtain additional information for the modelling of the return to isotropy. This mechanism has been studied very carefully in the physical space by Lumley & Newmann (1977) and more recently in spectral space by Cambon (1979); this author has attempted to extend to homogeneous (but non-isotropic) turbulence the eddy-damped quasi-normal approximation proposed by Orszag (1970) for the prediction of the spectrum of kinetic energy in isotropic turbulence.

2. On the influence of the nonlinear effects on the rotation of the principal axes of the Reynolds stress tensor

When the angle α between the principal axes of the two successive plane strains was different from both 0 and $\frac{1}{2}\pi$, it was observed that the principal axes of the Reynolds stress tensor, which were initially aligned with those of the first strain, had a strong

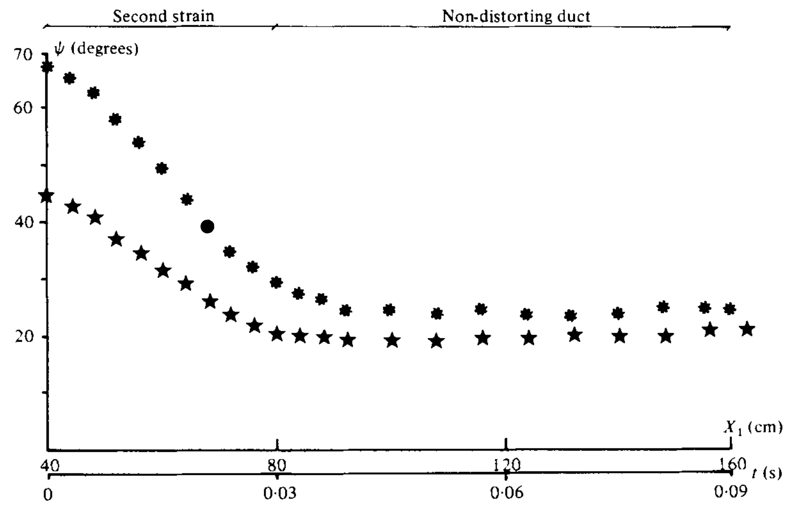

FIGURE 2. Evolution of the angle ϕ giving the position of the principal axes of the Reynolds stress tensor in the frame work (X_2, X_3) associated with the principal axes of the second strain.

tendency to become aligned with those of the second strain. It follows that the rate of rotation of the principal axes of the Reynolds stress tensor is different from zero when the second strain is applied. We can speculate whether this rate of rotation remains different from zero when the action of the second strain is suddenly suppressed. Such an evolution would give some information about the action of the nonlinear terms on this rate of rotation.

This nonlinear effect can of course be observed in the non-distorting duct placed after the second strain. The position of the principal axes of the Reynolds stress tensor is given by the angle ϕ between these axes and those of the second strain which are used as a galilean frame R_0 (figure 1) even when the second strain is no longer applied. The experiment was carried out for two values of α which were $\frac{1}{4}\pi$ and $\frac{3}{8}\pi$. It appears clearly in figure 2 that the angle ϕ remains constant in the non-distorting duct placed after the second strain. It follows then that in this experiment the nonlinear mechanisms associated with the self-interaction of the fluctuating motion have no significant effect on the rotation of the principal axes of the Reynolds stress tensor. It will be seen below that such a result gives information on the non-linear part of the pressure deformation correlations.

To underline the role of the rate of rotation of the principal axes of the Reynolds stress tensor which will be defined by the antisymmetric second-order tensor $\boldsymbol{\omega}$, the rate equations for the Reynolds stress tensor will be written with respect to its own principal axes. This new frame work R is of course non-galilean and has the rate of rotation $\boldsymbol{\omega}$ with respect to the galilean frame R_0. The tensors $\mathbf{P}^{NL}$ and $\boldsymbol{\epsilon}$ represent respectively the non-linear part of the pressure deformation correlations and the viscous term appearing in the equation of the Reynolds stress tensor. More precisely we can write

$$P_{ij}^{NL} = \overline{\frac{P}{\rho}\left(\frac{\partial u_i}{\partial X_j}+\frac{\partial u_j}{\partial X_i}\right)}, \tag{1}$$

the instantaneous fluctuating pressure being given by

$$-\frac{\Delta P}{\rho}=\frac{\partial u_i}{\partial X_j}\frac{\partial u_j}{\partial X_i}, \tag{2}$$

which is nonlinear with respect to the fluctuating velocity u_i, and

$$\epsilon_{ij}=-2\nu\overline{\frac{\partial u_i}{\partial X_K}\frac{\partial u_j}{\partial X_K}}. \tag{3}$$

The time derivatives in the frame works R and R_0 will respectively be termed:

$$\left.\frac{d}{dt}\right|_R;\quad \left.\frac{d}{dt}\right|_{R_0}.$$

In the non-distorting duct, the rate equation of the Reynolds stress tensor can be written:

$$\left.\frac{d}{dt}\overline{\mathbf{u}\otimes\mathbf{u}}\right|_{R_0}=\mathbf{P}^{NL}+\boldsymbol{\epsilon}. \tag{4}$$

However, it is easy to verify that

$$\left.\frac{d}{dt}\overline{\mathbf{u}\otimes\mathbf{u}}\right|_{R_0}=\left.\frac{d}{dt}\overline{\mathbf{u}\otimes\mathbf{u}}\right|_{R}+\boldsymbol{\omega}.\overline{\mathbf{u}\otimes\mathbf{u}}-\overline{\mathbf{u}\otimes\mathbf{u}}.\boldsymbol{\omega}. \tag{5}$$

Accordingly in the non-galilean frame work R we find that:

$$\left.\frac{d}{dt}\overline{\mathbf{u}\otimes\mathbf{u}}\right|_{R}=\overline{\mathbf{u}\otimes\mathbf{u}}.\boldsymbol{\omega}-\boldsymbol{\omega}.\overline{\mathbf{u}\otimes\mathbf{u}}+\mathbf{P}^{NL}+\boldsymbol{\epsilon}. \tag{6}$$

If $\overline{u^2_{(\alpha)}}$ are the principal values of the Reynolds stress tensor, we obtain in R the following expressions (without summation over the repeated indices):

$$\left.\begin{aligned}\frac{d}{dt}\overline{u^2_{(\alpha)}}&=P^{NL}_{\alpha\alpha}+\epsilon_{\alpha\alpha}\quad(\alpha=1,2,3),\\ 0\quad&=\omega_{\alpha\beta}(\overline{u^2_{(\beta)}}-\overline{u^2_{(\alpha)}})+P^{NL}_{\alpha\beta}+\epsilon_{\alpha\beta}\quad(\alpha\neq\beta).\end{aligned}\right\} \tag{7}$$

These six equations can be split into two parts: the first three parts give the evolution of the principal values of the Reynolds stress tensor and the other three give the three independent components of the antisymmetric tensor $\boldsymbol{\omega}$. In the present experiment, as shown in figure 2, it is found that $d\phi/dt$ is zero in the non-distorting duct, so that we can conclude:

$$\boldsymbol{\omega}=0, \tag{8}$$

which implies from (7):

$$P^{NL}_{\alpha\beta}+\epsilon_{\alpha\beta}=0. \tag{9}$$

Hence it follows that the right-hand side of equation (4), which is the tensor

$$P^{NL}_{ij}+\epsilon_{ij}=\overline{\frac{P}{\rho}\left(\frac{\partial u_i}{\partial X_j}+\frac{\partial u_j}{\partial X_i}\right)}-2\nu\overline{\frac{\partial u_i}{\partial X_K}\frac{\partial u_j}{\partial X_K}} \tag{10}$$

has the same principal axes as the Reynolds stress tensor. Such a result was not evident *a priori* since the nonlinear part of the pressure deformation correlation is a functional of the triple velocity correlations at two points M and M'. This functional is defined by

$$P_{ij}^{NL} = \frac{1}{4\pi}\cdot\int_{R^3}\left\{\frac{\overline{\partial^3 u_\rho u_m u_i'}}{\partial r_\rho \partial r_m \partial r_j} + \frac{\overline{\partial^3 u_\rho u_m u_j'}}{\partial r_\rho \partial r_m \partial r_i}\right\}\frac{d\mathbf{r}}{||\mathbf{r}||},$$

where $\mathbf{r} = \mathbf{MM}'$, and

$$\overline{u_\rho u_m u'_i}(\mathbf{r};t) = \overline{u_\rho(M_1 t)\, u_m(M,t)\, u_i(M',t)}.$$

These conclusions agree with the closure law proposed by Lumley & Newmann (1977) for the right-hand side of the equation (4) which they wrote as

$$\frac{d}{dt}\overline{u_i u_j}\bigg|_{R_0} = \left\{\overline{\frac{P}{\rho}\left(\frac{\partial u_i}{\partial X_j}+\frac{\partial u_j}{\partial X_i}\right)} - \left(2\gamma\,\overline{\frac{\partial u_i}{\partial X_K}\frac{\partial u_j}{\partial X_K}} - \tfrac{2}{3}\bar{\epsilon}\delta_{ij}\right)\right\} - \tfrac{2}{3}\bar{\epsilon}\delta_{ij}, \tag{11}$$

$\bar{\epsilon}$ being the rate of dissipation.

They denoted the term in brackets by $-\epsilon\Phi_{ij}$ so that they should obtain

$$\frac{d}{dt}\overline{u_i u_j}\bigg|_{R_0} = -\bar{\epsilon}\Phi_{ij} - \frac{2}{3}\bar{\epsilon}\delta_{ij}; \tag{12}$$

Here Φ_{ij} appears as a dimensionless traceless second-order symmetric tensor which they supposed to be an isotropic function of the Reynolds number Re_L (L being of the same order of magnitude as an integral scale) and of the tensor $\mathbf{b}$ defined by

$$b_{ij} = \frac{\overline{u_i u_j}}{\overline{q^2}} - \frac{\delta_{ij}}{3}; \tag{13}$$

$\overline{q^2}$ is the trace of the Reynolds stress tensor.

Putting

$$\mathrm{II} = b_{iK} b_{Ki}, \quad \mathrm{III} = b_{iK} b_{Kj} b_{ji} \tag{14}$$

Lumley & Newmann obtain by means of the representation theorems

$$\Phi_{ij} = \beta(\mathrm{II}, \mathrm{III}, Re_L)\, b_{ij} + \gamma(\mathrm{II}, \mathrm{III}, Re_L)\,(b_{iK} b_{Kj} - \tfrac{1}{3}\mathrm{II}\,\delta_{ij}). \tag{15}$$

It then follows that the tensor Φ_{ij} which in fact is linked to the right-hand side of equation (4) by

$$-\bar{\epsilon}\Phi_{ij} = P_{ij}^{NL} + \epsilon_{ij} - \tfrac{2}{3}\bar{\epsilon}\delta_{ij}, \tag{16}$$

has the same principal axes as both the tensor b_{ij} and the Reynolds stress tensor, which agrees well with the author's own experimental results.

As in this particular experiment the turbulence is strongly influenced by sudden changes in the behaviour of the mean flow, it is reasonable to admit that the preceding result can be extended to more general cases not exhibiting so rapid change in the mean flow distribution. This result appears as an aspect of the 'fading memory' of the turbulent motion which is often used as a basic hypothesis in the closure methods in physical space and in particular for obtaining a relation such as (15).

When the Reynolds number is large enough, we can write

$$\epsilon_{ij} = -2\nu\,\overline{\frac{\partial u_i}{\partial X_K}\frac{\partial u_j}{\partial X_K}} \approx -\tfrac{2}{3}\bar{\epsilon}\delta_{ij},$$

and hence the relation (10) shows that the tensor P_{ij}^{NL} must have the same principal axes as the Reynolds stress tensor.

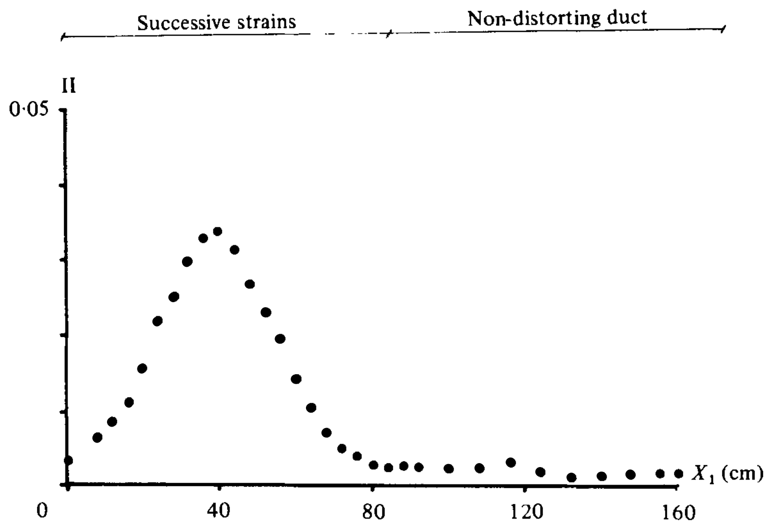

FIGURE 3. Evolution of the invariant II when two successive plane strains are of opposite sign, $\alpha = \frac{1}{2}\pi$.

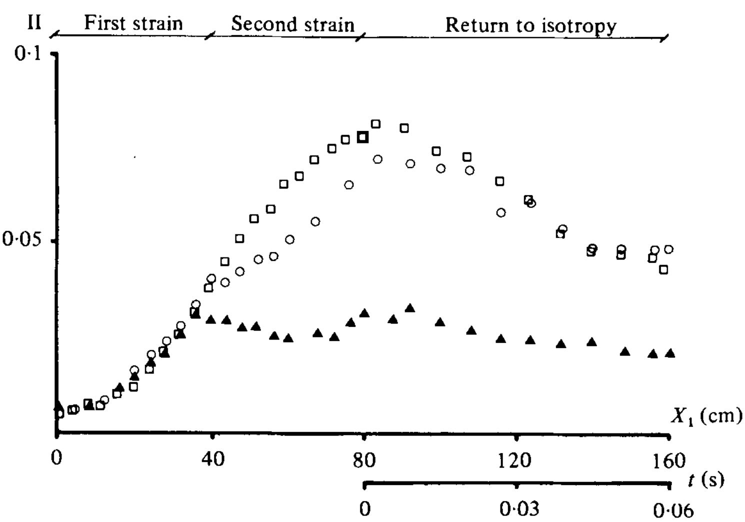

FIGURE 4. Evolution of the invariant II for three values of the angle α between the principal axes of the two successive plane strains.

3. On the influence of the sign of the invariant III on the dynamics of the return to isotropy

Another aspect of the self-interaction of the fluctuating motion is the well-known tendency to return to an isotropic state, which means that the principal values of the Reynolds stress tensor tend to become equal as time increased. This has been observed in several experiments and in particular in those of Mills & Corrsin (1959), Uberoi (1956), and more recently in those of Tücker & Reynolds (1968). Some numerical

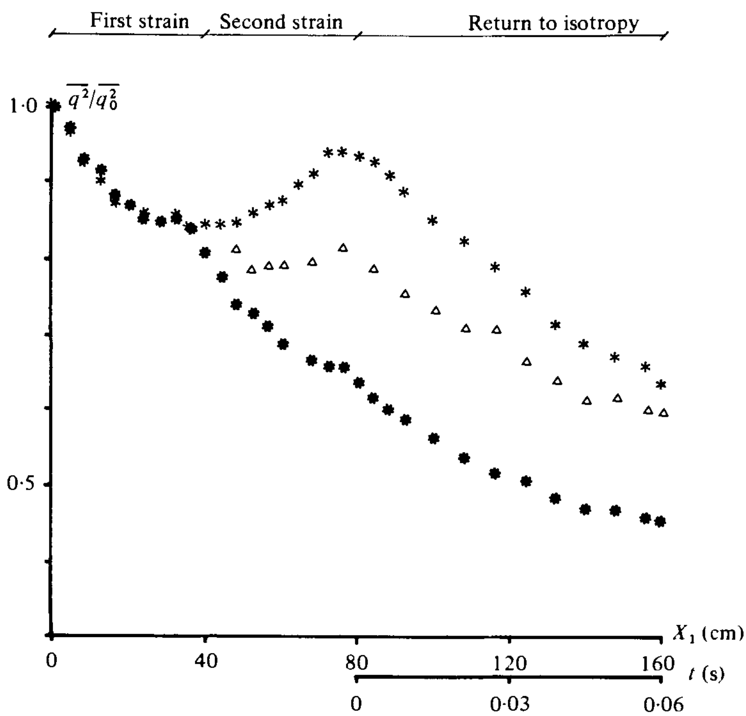

FIGURE 5. Evolution of the ratio $\overline{q^2}/\overline{q_0^2}$ for three values of the angle α between the principal axes of the two successive plane strains: *, $\alpha = 0$; $\triangle$, $\alpha = \frac{1}{4}\pi$; *, $\alpha = \frac{3}{8}\pi$.

simulations carried out by Schumann & Patterson (1978) show the same phenomenon. The immediate consequence is that the different components of the tensor **b** defined by (13) and the invariant II introduced in (14) tends to zero. Starting from the equation (12), the rate equation of II can be written

$$\frac{d\mathrm{II}}{dt} = -2\frac{\bar{\epsilon}}{\overline{q^2}}[\Phi_{ij}b_{ij} - 2\mathrm{II}]. \tag{17}$$

A characteristic time of return to isotropy can then be defined as:

$$\tau_r = -\mathrm{II}\left(\frac{d\mathrm{II}}{dt}\right)^{-1}, \tag{18}$$

which will be compared to the time of decay

$$\tau_D = -\overline{q^2}\left(\frac{d\overline{q^2}}{dt}\right)^{-1}.$$

As in the return to isotropy we have

$$\frac{d\overline{q^2}}{dt} = -2\bar{\epsilon}.$$

The line τ_D can also be written

$$\tau_D = \frac{\overline{q^2}}{2\bar{\epsilon}}, \tag{19}$$

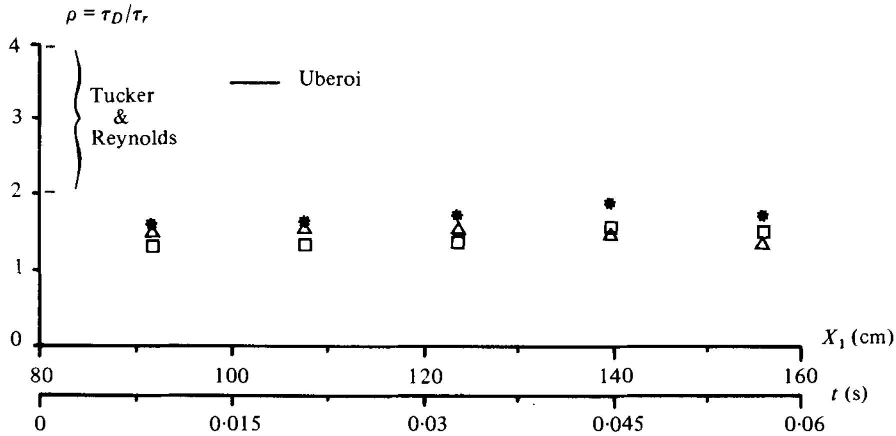

FIGURE 6. Evolution of the ratio ρ of the characteristic time of decay τ_D to the characteristic time of return to isotropy τ_r during the return to isotropy for three values of the angle α between the principal axes of the two successive plane strains: *, $\alpha = 0$; △, $\alpha = \frac{1}{4}\pi$; □, $\alpha = \frac{3}{8}\pi$.

Experiment	Interval of variation of III		Interval of variation of II		Interval of variation of Re_L		Interval of variation of ρ	
Mills–Corrsin (1959)	$-5{\cdot}3 \times 10^{-3}$	$-0{\cdot}34 \times 10^{-3}$	0·055	0·008	10	40	2	6
Uberoi (1956)	$-7{\cdot}15 \times 10^{-3}$	$-1{\cdot}3 \times 10^{-5}$	0·067	0·001	43	34	3·5	4·5
Tücker & Reynolds (1968)	$-3{\cdot}2 \times 10^{-3}$	$-4{\cdot}5 \times 10^{-4}$	0·081	0·0210	310	330	4·5	2
Present experiment (when $\alpha = 0$)	$+4{\cdot}2 \times 10^{-3}$	$+2{\cdot}6 \times 10^{-3}$	0·078	0·0420	400	450	1·5	1·9
Schumann & Patterson (1978) (run A2)	$-1{\cdot}4 \times 10^{-2}$	$-5{\cdot}5 \times 10^{4}$	0·109	0·012	33	10	2·4	0·8

TABLE 1.

so that the ratio ρ of these two fundamental times appears as given by

$$\rho = \frac{\tau_D}{\tau_r} = \frac{1}{\mathrm{II}}\Phi_{ij}b_{ij} - 2, \tag{20}$$

which can be considered as an important parameter of the dynamics of the return to isotropy.

In the author's experiments, different levels of anisotropy, that is to say different values of the invariant II in the initial section of the non-distorting duct, can be obtained without difficulty by changing the value of the angle α between the principal axes of the two successive plane strains. The highest value of II is obtained when α equals zero and the lowest value when α equals $\frac{1}{2}\pi$. In the second case it has been shown (Gence & Mathieu 1979) that the fluctuating motion is acted upon by the second strain to return to an isotropic state in a finite time, so that II should possess a value slightly different from zero at the entrance of the non-distorting duct, and hence, no sensible effect of return to isotropy can be observed (figure 3).

To study the decay of the invariant II in the author's experiment, the three values, 0, $\frac{1}{4}\pi$ and $\frac{3}{8}\pi$ were considered for α. Its evolution along the axis of the non-distorting duct is shown in figure 4 and that of $\overline{q^2}$ is given in figure 5. The ratio ρ can easily be derived from these results, and the figure 6 indicates that for the three values of α corresponding to three different initial values of II, the characteristic time of decay is approximately 1·7 times greater than the time of return to isotropy. A comparison of the values of ρ obtained in the different experiments dealing with that physical situation is given in table 1. It appears that all these values are greater than in the present case and that the most interesting example which can be compared with the author's experiment when $\alpha = 0$ is the experiment of Tücker & Reynolds (1968) where the initial value of II is very close to the values obtained in the present situation. Another interesting point which justifies such a comparison is that the Reynolds number linked to the fluctuating motion, which, in agreement with Lumley & Newmann (1977), can be written

$$Re_L = \frac{(\overline{q^{\frac{2}{3}}})^{\frac{1}{2}} L}{\nu} = \frac{(\overline{q^2})^2}{9\bar{\epsilon}\nu}, \tag{21}$$

is approximately of the same order of magnitude in the two experiments. To understand the observed difference between the two values of ρ, it is necessary to consider a third parameter which appears in the analysis of Lumley & Newmann (1977). They attempted to close the following set of equations:

$$\left.\begin{aligned} \frac{d}{dt}\overline{u_i u_j} &= -\bar{\epsilon}\Phi_{ij} - \tfrac{2}{3}\bar{\epsilon}\,\delta_{ij}, \\ \frac{d\bar{\epsilon}}{dt} &= -\frac{(\bar{\epsilon})^2}{\overline{q^2}}\psi, \end{aligned}\right\} \tag{22}$$

where Φ_{ij} is defined by (16) and the dimensionless unknown ψ by

$$-\frac{(\bar{\epsilon})^2}{\overline{q^2}}\psi = -2\nu\overline{\frac{\partial u_i}{\partial X_K}\frac{\partial u_j}{\partial X_K}\frac{\partial u_i}{\partial X_j}} - 2\nu^2\overline{\frac{\partial^2 u_i}{\partial X_K \partial X_j}\frac{\partial^2 u_i}{\partial X_K \partial X_j}}. \tag{23}$$

Assuming that the turbulence has a fading memory, these authors closed the set (22) considering that Φ_{ij} and ψ are functions of the Reynolds stress tensor, the rate of dissipation $\bar{\epsilon}$ and the kinematic viscosity. They obtained then the expression (15) for Φ_{ij} and for ψ they wrote

$$\psi = \psi(\mathrm{II}, \mathrm{III}, Re_L),$$

the different variables are defined by (13) and (14). It appears clearly that the invariant III must be taken into account to describe the dynamics of the return to isotropy. In particular (15) and (20) lead to

$$\rho = \beta(\mathrm{II}, \mathrm{III}, Re_L) - 2 + \frac{\mathrm{III}}{\mathrm{II}}\gamma(\mathrm{II}, \mathrm{III}, Re_L). \tag{24}$$

In all the known experiments of return to isotropy the invariant III is a negative quantity but it is positive in the present experiment. It is possible to give a simple physical interpretation of this sign in the case of axisymmetric turbulence as indicated

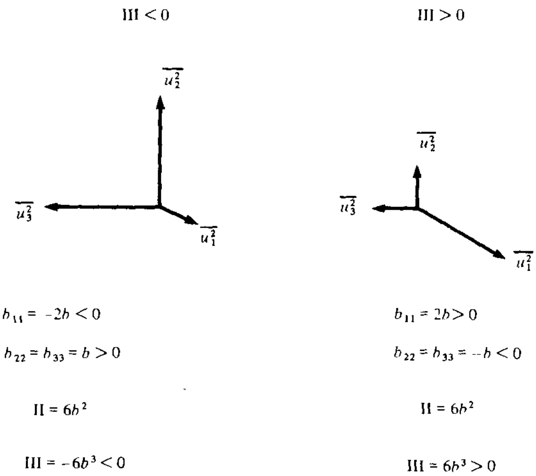

FIGURE 7. Scheme giving the physical meaning of the sign of the invariant III in the particular case of axisymmetrical turbulence.

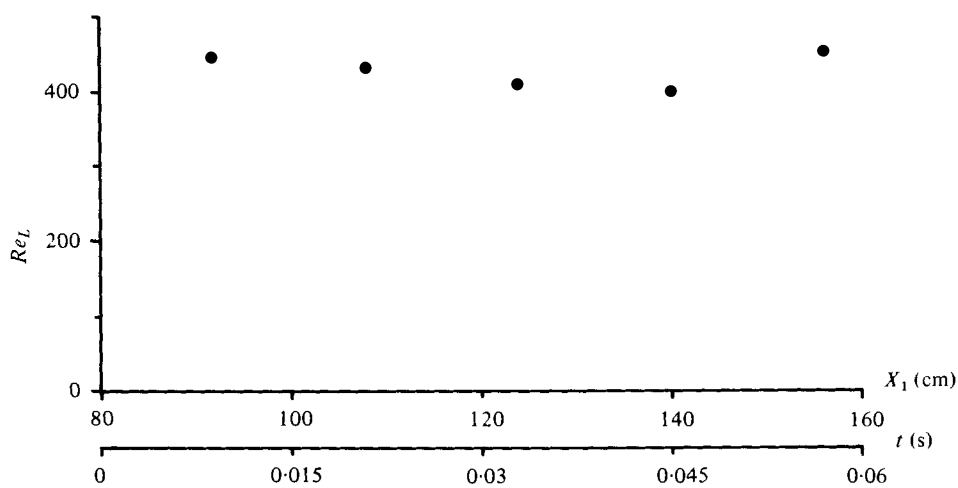

FIGURE 8. Values of the turbulent Reynolds number Re_L during the return to isotropy in the case when the two successive plane strains are identical ($\alpha = 0$).

in figure 7. It can be verified that III is a positive quantity when the component of the Reynolds stress tensor along the symmetry axis is greater than the two others and is a negative quantity in the opposite case.

Comparing the experiments of Tücker & Reynolds with the present configuration when α equals zero, we conclude that for the corresponding values of II and Re_L, the function ρ (II, III, Re_L) must be a decreasing function of III. It is simple to verify that our experimental results are in agreement with the hypothesis

$$\left.\begin{aligned}\gamma(\mathrm{II},\mathrm{III},Re_L) &= 0\\ \beta(\mathrm{II},\mathrm{III},Re_L) &= 2+\beta_0(Re_L)+\beta_1(Re_L)\,\mathrm{II}+\beta_2(Re_L)\,\mathrm{III}.\end{aligned}\right\} \tag{25}$$

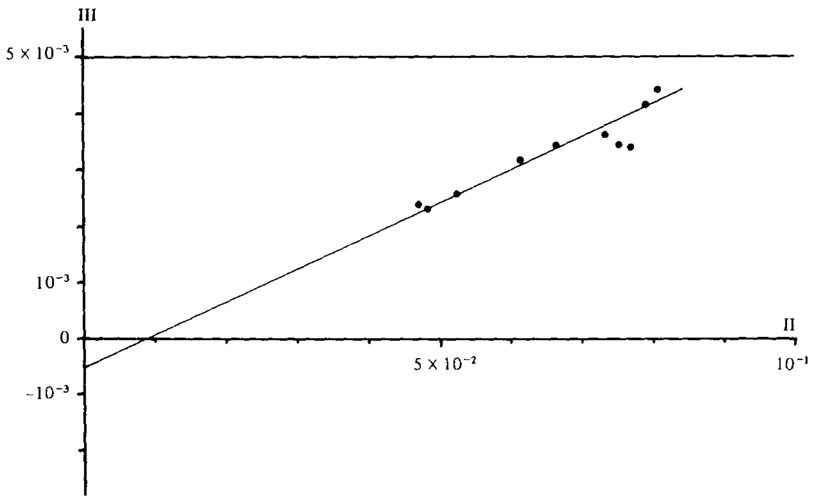

FIGURE 9. The linear relation between the invariants II and III in the return to isotropy obtained in the case when the two successive plane strains are identical ($\alpha = 0$).

From (24) it follows indeed that it can be written as

$$\rho = \beta_0(Re_L) + \beta_1(Re_L)\,\mathrm{II} + \beta_2(Re_L)\,\mathrm{III}, \tag{26}$$

and the authors find that ρ and Re_L (figure 8) are nearly constant so that the relation between III and II given by (26) should be a straight line, which is the case under consideration as shown in figure 9. Moreover this curve indicates that if $\beta_1(Re_L)$ is positive for that Reynolds number, $\beta_2(Re_L)$ must be negative so that ρ appears as a decreasing function of III. To go further and to give accurate conclusions concerning the influence of the sign of the invariant III, additional experiments of return to isotropy must be carried out, in which the values of II and Re_L and the sign of III can be modified systematically.

4. Conclusion

The experiment has shown that during the return to isotropy the orientation of the principal axes of the Reynolds stress tensor does not change even in a situation where the history of the turbulence has been influenced by sudden changes in the mean velocity gradient. Such a result implies in particular that the nonlinear part of the pressure deformation correlation is a tensor which has the same principal axes as the Reynolds stress tensor, when the Reynolds number tends to infinity.

Moreover, it appears that the dynamics of the return to isotropy is influenced by the invariant III defined as the trace of the cube of the tensor **b** as has already been argued by Lumley & Newmann (1977). In all the previous experiments of return to isotropy this invariant is negative but it is positive in the present situation. A comparison with the study of Tücker & Reynolds (1968) seems to indicate that when III is positive, the return to isotropy is slower than in the case when it is negative.

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